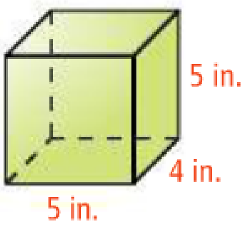
Chapter 11 - Day2
Surface Area & Volume

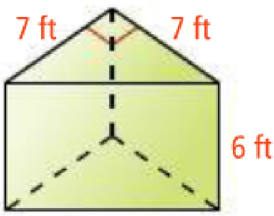
Lesson objectives: to discover the properties of surface area and volume and how to apply those properties to solve problems.

What is the surface area of each prism?

1.

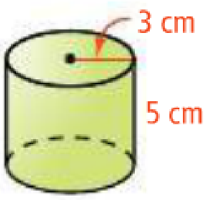


2.

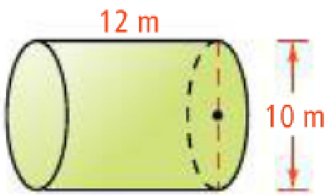


What is the surface area of each cylinder?

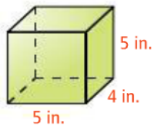
3.

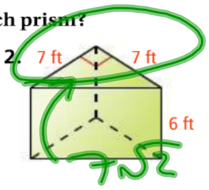


4.

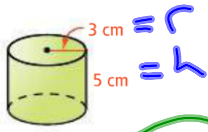


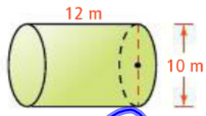
What is the surface area of each prism?

1.


2.


What is the surface area of each cylinder?

3.


4.


1. 130 in.^{2}

2. $(133 + 42\sqrt{2})$ ft^2 or about 192.4 ft^2

3. 48π cm^2 or about 150.8 cm^2

4. 170π m^2 or about 534.1 m^2

③ $LA = ph$

$2\pi r * h$

$LA = 2\pi(3)(5)$

$= 30\pi$

$SA = 30\pi + (\pi r^2)(2)$

$= 30\pi + 18\pi$

$= 48\pi$

④ $LA = ph$ $2\pi r * h$

$2\pi(5)(12) = 120\pi$

$\pi r^2 = \pi(25)$

$B = 2(25)\pi$

$= 50\pi$

$LA + 2B$

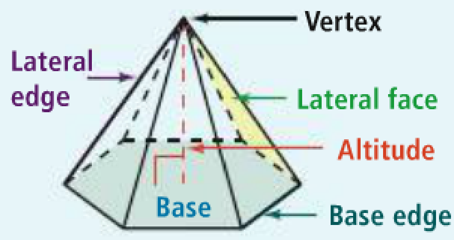
$SA = 120\pi + 50\pi$

$= 170\pi$

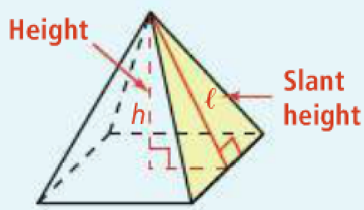
A **pyramid** is a polyhedron in which one face (the **base**) can be any polygon and the other faces (the **lateral faces**) are triangles that meet at a common vertex (called the **vertex** of the pyramid).

You name a pyramid by the shape of its base. The **altitude** of a pyramid is the perpendicular segment from the vertex to the plane of the base. The length of the altitude is the **height** h of the pyramid.

A **regular pyramid** is a pyramid whose base is a regular polygon and whose lateral faces are congruent isosceles triangles. The **slant height** ℓ is the length of the altitude of a lateral face of the pyramid.



Hexagonal pyramid



Square pyramid

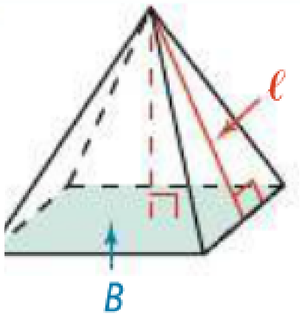
Theorem 11-3 Lateral and Surface Areas of a Pyramid

The lateral area of a regular pyramid is half the product of the perimeter p of the base and the slant height ℓ of the pyramid.

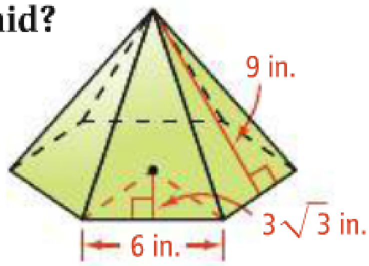
$$\text{L.A.} = \frac{1}{2}p\ell$$

The surface area of a regular pyramid is the sum of the lateral area and the area B of the base.

$$\text{S.A.} = \text{L.A.} + B$$



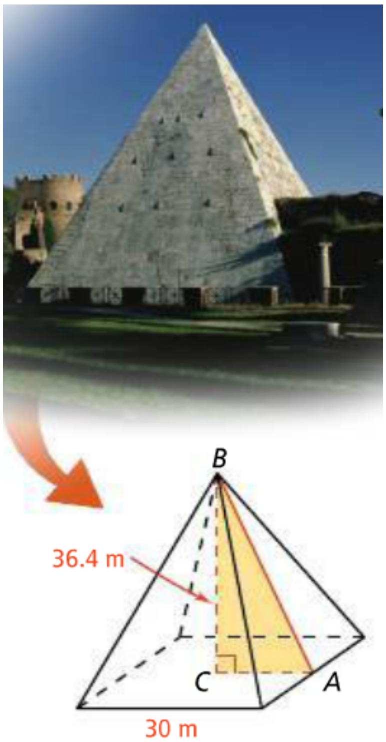
What is the surface area of the hexagonal pyramid?



$\text{S.A.} = \text{L.A.} + B$	Use the formula for surface area.
$= \frac{1}{2}p\ell + \frac{1}{2}ap$	Substitute the formulas for L.A. and B.
$= \frac{1}{2}(36)(9) + \frac{1}{2}\left(3\sqrt{3}\right)(36)$	Substitute.
≈ 255.5307436	Use a calculator.

The surface area of the pyramid is about 256 in.^{2}.

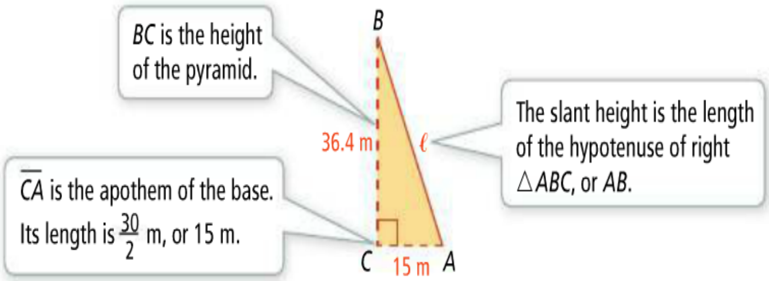
Social Studies The Pyramid of Cestius is located in Rome, Italy. Using the dimensions in the figure below, what is the lateral area of the Pyramid of Cestius? Round to the nearest square meter.



Step 1 Find the perimeter of the base.

$p = 4s$	Use the formula for the perimeter of a square.
$= 4 \cdot 30$	Substitute 30 for s.
$= 120$	Simplify.

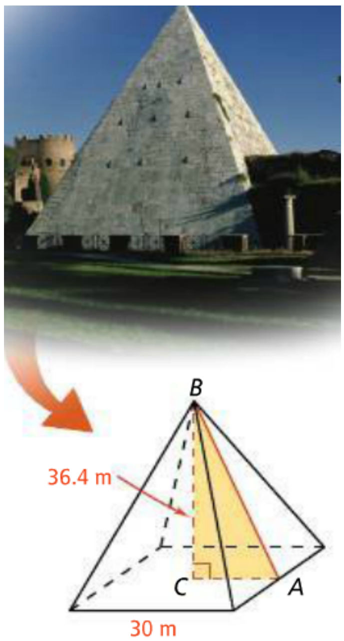
Step 2 Find the slant height of the pyramid.



$$\ell = \sqrt{CA^2 + BC^2}$$ Use the Pythagorean Theorem.

$$= \sqrt{15^2 + 36.4^2}$$ Substitute 15 for CA and 36.4 for BC.

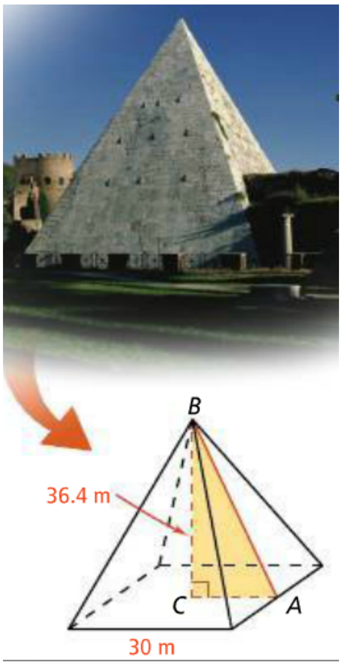
$$= \sqrt{1549.96}$$ Simplify.



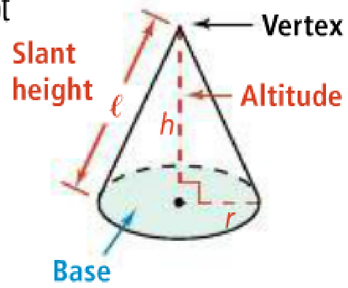
Step 3 Find the lateral area of the pyramid.

$$L.A. = \frac{1}{2}p\ell \quad \text{Use the formula for lateral area.}$$
$$= \frac{1}{2}(120)\sqrt{1549.96} \quad \text{Substitute 120 for } p \text{ and } \sqrt{1549.96} \text{ for } \ell.$$
$$\approx 2362.171882 \quad \text{Use a calculator.}$$

The lateral area of the Pyramid of Cestius is about 2362 m^2.

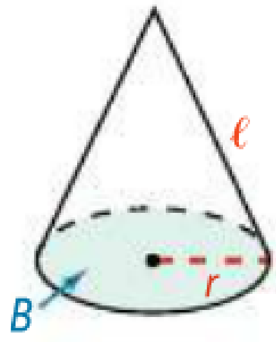


Like a pyramid, a **cone** is a solid that has one base and a vertex that is not in the same plane as the base. However, the **base** of a cone is a circle. In a **right cone,** the **altitude** is a perpendicular segment from the **vertex** to the center of the base. The **height** h is the length of the altitude. The **slant height** ℓ is the distance from the vertex to a point on the edge of the base. In this book, you can assume that a cone is a right cone unless stated or pictured otherwise.



The **lateral area** is half the circumference of the base times the slant height. The formulas for the lateral area and **surface area** of a cone are similar to those for a pyramid.

Theorem 11-4 Lateral and Surface Areas of a Cone



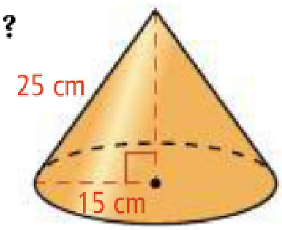
The lateral area of a right cone is half the product of the circumference of the base and the slant height of the cone.

$$\text{L.A.} = \frac{1}{2} \cdot 2\pi r \cdot \ell, \text{ or L.A.} = \pi r \ell$$

The surface area of a cone is the sum of the lateral area and the area of the base.

$$\text{S.A.} = \text{L.A.} + B$$

What is the surface area of the cone in terms of π?

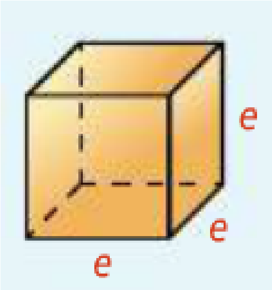


$\text{S.A.} = \text{L.A.} + B$	Use the formula for surface area.
$= \pi r \ell + \pi r^2$	Substitute the formulas for L.A. and B.
$= \pi(15)(25) + \pi(15)^2$	Substitute 15 for r and 25 for ℓ.
$= 375\pi + 225\pi$	Simplify.
$= 600\pi$	

The surface area of the cone is 600π cm^2.

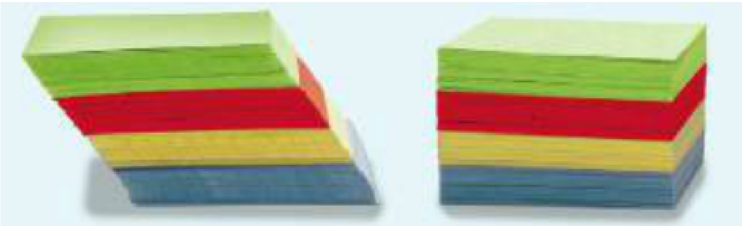
Volume

Volume is the space that a figure occupies. It is measured in cubic units such as cubic inches (in.^{3}), cubic feet (ft^3), or cubic centimeters (cm^3). The volume V of a cube is the cube of the length of its edge e, or $V = e^3$.



You can find the volume of a prism or a cylinder when you know its height and the area of its base.

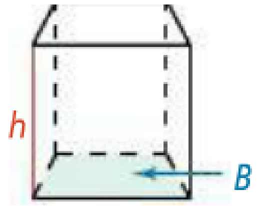
Both stacks of paper below contain the same number of sheets.



Theorem 11-6 Volume of a Prism

The volume of a prism is the product of the area of the base and the height of the prism.

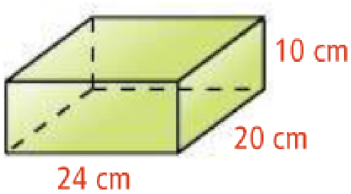
$$V = Bh$$



Note the use of "B" versus "b"!

Finding the Volume of a Rectangular Prism

What is the volume of the rectangular prism at the right?



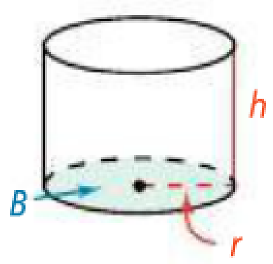
$V = Bh$ Use the formula for the volume of a prism.

$= 480 \cdot 10$ The area of the base B is $24 \cdot 20$, or $480\ \text{cm}^2$, and the height is 10 cm.

$= 4800$ Simplify.

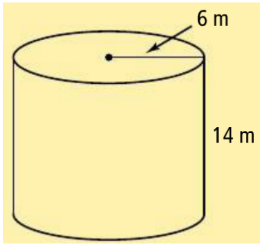
Theorem 11-7 Volume of a Cylinder

The volume of a cylinder is the product of the area of the base and the height of the cylinder.



$$V = Bh \quad \text{(or)} \quad V = \pi r^2 h$$

Find the volume of the cylinder in terms of π.

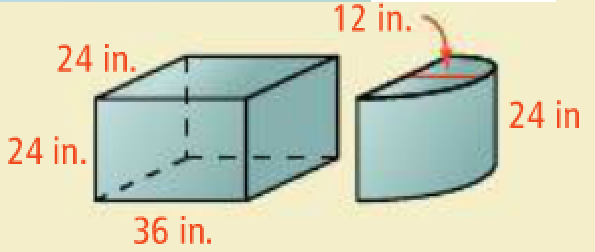


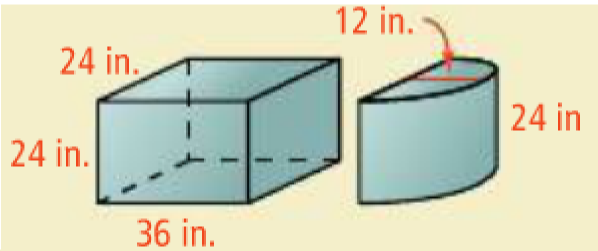
$$V = Bh$$

ANSWER 504π m^2

A **composite space figure** is a three-dimensional figure that is the combination of two or more simpler figures. You can find the volume of a composite space figure by adding the volumes of the figures that are combined.

The length of the prism is the total length minus the radius of the cylinder. The radius of the cylinder is half the width of the prism.





$V_1 = Bh$

$= (24 \cdot 36)(24)$

$= 20{,}736$

$V_2 = \frac{1}{2}\pi r^2 h$

$= \frac{1}{2}\pi(12)^2(24)$

≈ 5429

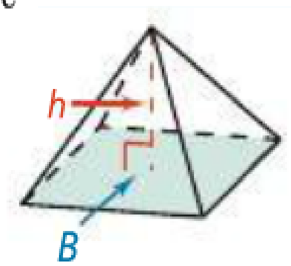
$20{,}736 + 5429 = 26{,}165$

The approximate volume of the aquarium is $26{,}165 \text{ in.}^3$.

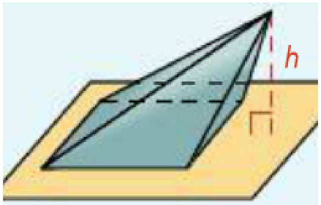
Theorem 11-8 Volume of a Pyramid

The volume of a pyramid is one third the product of the area of the base and the height of the pyramid.

$$V = \frac{1}{3}Bh$$



This formula works for *any* pyramid, whether right or oblique.



Oblique pyramid